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Title: ACTIVE CONTROL OF PANEL VIBRATIONS
INDUCED BY A BOUNDARY LAYER FLOW

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I. INTRODUCTION

In recent years, active and passive control of sound and vibration in aeroelastic structures have received a great deal of attention due to many potential applications to aerospace and other industries. There exists a great deal of research work done in this area. Recent advances in the control of sound and vibration can be found in the several conference proceedings. In this report we will summarize our research findings supported by the NASA grant NAG-1-1175.

The problems of active and passive control of sound and vibration has been investigated by many researchers for a number of years. However, few of the articles are concerned with the sound and vibration with flow-structure interaction. Experimental and numerical studies on the coupling between panel vibration and acoustic radiation due to flow excitation have been done by Maestrello and his associates at NASA/Langley Research Center. Since the coupled system of nonlinear partial differential equations is formidable, an analytical solution to the full problem seems impossible. For this reason, we have to simplify the problem to that of the nonlinear panel vibration induced by a uniform flow or a boundary-layer flow with a given wall pressure distribution. Based on this simplified model, we have been able to study the control and stabilization of the nonlinear panel vibration, which have not been treated satisfactorily by other authors. The vibration suppression will clearly reduce the sound radiation power from the panel. The major research findings will be presented in the next three sections. In Section II we shall describe our results on the boundary control of nonlinear panel vibration, with or without flow excitation. Section III is concerned with active control of the vibration and sound radiation from a nonlinear elastic panel. A detailed description of our work on the parametric vibrational control of nonlinear elastic panel will be presented in Section IV. This paper will be submitted to the Journal of Acoustic Society of America for publication.

II. BOUNDARY CONTROL OF NONLINEAR PANEL VIBRATION

This work is concerned with the analytical study of a stabilization scheme to suppress the large amplitude panel vibration by means of boundary damping: that is, by introducing extraneous dissipative boundary conditions, an otherwise unstable system will achieve its global and exponential stability. To be specific, we consider the panel vibration to be governed by a nonlinear beam equation, with or without aerodynamic forcing, subject to the clamped-free boundary conditions: that is, the left end of the panel is clamped and the right end is free to move. Without any control, the compressive force and the aerodynamic loading may be time-dependent. To stabilize the system, we apply

to the free edge a set of boundary controls, which consists of the combination of a vertical force, a bending moment and a tensile force. By means of the energy method and some mathematical inequalities, the boundary stabilization of the model problem for a vibrating nonlinear elastic panel was treated analytically. In general, the panel is subject to a compressive in-plane loading combined with an aerodynamic forcing. Without any control, the panel may buckle and would oscillate due to flow induced instability. To stabilize the panel, a set of boundary controls was introduced as the combination of a bending moment, a vertical point force and a tensile force applied to the free edge. Two cases, corresponding to the absence and the presence of an aerodynamic loading, were treated separately.

Without the flow, this is the case of free vibration. Even though the energy of the uncontrolled system is conserved, with an initial disturbance, the system may buckle or sustain persistent large amplitude oscillation. To render the energy an exponential decay, it was found sufficient to apply a tensile force, if necessary, to reduce the net force to a subcritical level and, at the same time, to introduce boundary damping. The damping mechanism consists of frictional forces and torque, which are linearly proportional to the transverse and the angular velocities of the right edge, respectively. Therefore, if the compressive force is subcritical, the passive control in the form of boundary damping suffices to stabilize the system. In an analogous situation, the result seems to be in agreement with the experimental evidence that boundary damping is effective in suppressing the panel vibration. In the presence of the unsteady flow and a compressive force, the panel may buckle or flutter. If the flow velocity is oscillatory and decays rapidly, it is possible to stabilize the panel by applying a time-varying tensile force together with boundary damping as before. However, for a slowly varying compressive force and, at the same time, with a small flow parameter the system can be stabilized as in the free-vibration case. For the purpose of illustration, several numerical examples were provided. In what follows, a few comments are in order.

- (1) The paper deals with a model problem which may not be realistic in practice. However, at least qualitatively, the analytical results support the experimental evidence that the passive control by boundary damping is effective in suppressing panel vibration.
- (2) As in the stability analysis of an ordinary differential equation, we set the flow pressure equal to zero, because one is interested only in the stability of the equilibrium solution due to an initial perturbation. In the presence of the pressure fluctuation, the system vibration may be suppressed by a distributed control. This will be discussed in the next section.
- (3) The theoretical approach based on the energy method and some mathematical inequalities, when applicable, is rather powerful. However, it certainly has its limitations. In general one needs both the analytical and numerical techniques to tackle the difficult the flow-structure interaction problems.

The results of this research and related work are summarized in our papers [1]-[3].

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III. ACTIVE CONTROL OF SOUND AND VIBRATION

The main purpose of this work is to study, based on a nonlinear panel model, the effectiveness of the active control to suppress the panel vibration and sound radiation induced by the unsteady pressure fluctuation. The control consists of a distributed force applied normally to one side of the wall. For simplicity, the flexible panel is assumed to be hinged to the rigid plates at both ends. The coupled equations governing the nonlinear panel vibration and acoustic radiation problem were given. For the optimality criterion, a time-average energy or objective functional was introduced to measure the performance in controlling the vibration and sound radiation. By applying the variational method, we derived the optimality equation for the control force distribution which is coupled with the controlled equations of motion. By using an eigenfunction expansion, the modal control problem was formulated. The truncated modal control problem was solved numerically by the shooting method for a two-point boundary value problem in the time domain. The numerical results were obtained to demonstrate the effectiveness of active control.

The main results of this research are summarized as follows.

- (1) For the control of panel vibration, given the control objective function, the optimal control can be found in the form of an external pressure applied to the wall. We derived the partial differential equation for the optimal control which is coupled to the equation of motion.
- (2) For the given objective function, it is possible to derive the optimality system for the control of sound radiation governed by the wave equation.
- (3) In both cases, the optimality system consists of a coupled nonlinear boundary value problem in space and time.
- (4) In the case of truncated modal control, the optimality system yields a two-point boundary value problem for a finite set of nonlinear ordinary differential equations.
- (5) The truncated optimality system for modal control was solved numerically. The results show the following: (i) for linear panel vibration, the control is highly effective and can almost completely eliminate the vibration over a short time horizon. (ii) In contrast, a non-linear panel, in general, responds less sensitively to the active control. The control is more effective at lower vibration frequencies and with weak nonlinearity. For fixed nonlinearity, the

effectiveness of control diminishes as the frequencies increase, and eventually the system loses control completely. (iii) By applying the optimal vibration control, the sound radiation intensity can also be reduced significantly.

The research is related to the numerical studies performed by Maestrello and some of his collaborators [1], [2]. The results of our work can be found in the published paper [3].

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IV. VIBRATIONAL CONTROL OF NONLINEAR PANEL

1. Introduction.

The problem under consideration is the stabilization of the nonlinear panel oscillation by an active control with a vibrational actuator. This work was motivated by the recent experimental investigations of the second author (Maestrello [1], [2]), who demonstrated clearly that the vibrational control could be an effective means of stabilizing the boundary-layer flow as well as the panel vibration. This paper will offer a general method of vibrational control and its application to the problem involving a nonlinear elastic panel excited by the periodic wall-pressure fluctuation in a boundary-layer flow.

The general principle of active control, the vibrational control in particular, is to introduce an action which affects a change in the behavior of a dynamical system in a desirable manner. In the boundary layer transition control [1], the periodic heating and cooling of the wall induce a parametric vibration of the fluid viscosity which, in turn, stabilizes the flow. In the case of panel vibration [2], a properly added vibrational force with the same forcing frequency may result in suppressing the subharmonic oscillations. (see section 2). The suppression of subharmonics, of course, has the implication of controlling the chaotic motion.

The main idea of vibrational control stems from the fact that an inverted pendulum can be stabilized at its upper equilibrium position when the lower suspension point executes a rapid vertical vibration. (see e.g. [3]). Based on this idea, a general principle of vibrational control was proposed, notably by Meerkov [4], to stabilize the equilibrium points of some finite-dimensional linear systems. Application of this principle to reactor dynamics was done by Bellman et al [5]. By contrast, in this paper, we will extend this control principle to stabilize the periodic motions of infinite-dimensional systems, instead of equilibrium points in finite dimensions. In addition to the high-frequency parametric vibrational control used in [4,5], a vibrational force with the same forcing frequency will be required. Unlike the usual feedback or feed-forward control, the vibrational control does not need accurate measurement of the system inputs and outputs and can be implemented much more easily, especially for an infinite-dimensional system under consideration.

In this paper we consider the panel vibration which satisfies the initial boundary value problem for the nonlinear beam equation [6]:

$$\begin{cases} m\partial_t^2 w + c\partial_t w - [Q + N(t)]\partial_x^2 w + D\partial_x^4 w = \Delta p(t, x), 0 < x < l \\ w(t, 0) = w(t, l) = 0, \partial_x^2 w(t, 0) = \partial_x^2 w(t, l) = 0, \\ w(0, x) = w_0(x), \partial_t w(0, x) = w_1(x). \end{cases} \quad (1.1)$$

Here w denotes the transverse deflection; $\partial_t, \partial_x \dots$ are the partial differentiations in $t, x \dots$; the positive constants in c and D represent the unit mass, the damping coefficient and the bending stiffness of the panel, respectively. The axial force Q is positive or negative according to the force being tensile or compressive. The large panel deflection introduces an additional tension $N(t)$ given by

$$N(t) = b \int_0^l |\partial_x w(t, x)|^2 dx, \quad (1.2)$$

where b is an elastic constant. The forcing term Δp denotes the pressure difference across the panel surfaces. The homogeneous boundary conditions mean that the panel is simply supported, and the initial data w_0 and w_1 are given. Suppose that, without any control, the periodic solution of equation (1.1) excited by the pressure Δp is unstable. Our problem is to stabilize the panel oscillation by applying an appropriate control in the form of vibrational forces added to the axial force and the

pressure Δp .

The paper is organized as follows. To illustrate the basic ideas involved, in section 2, we consider the control of the Duffing equation, for which the response characteristics to a time-harmonic excitation is well known. The feasibility of the vibrational control can be discussed geometrically by referring to the response curves. Since the applicability of vibrational control is not limited to the structure dynamics, in section 3, a general method of vibrational control for a class of nonlinear evolution equations is presented. For a given unstable periodic solution, the control strategy is to shift the Liapunov exponent r of the vibrational equation to the negative half-line so that the corresponding periodic solution becomes stable. This method is applied to the nonlinear panel vibration problem satisfying equation (1.1). For weak nonlinearity, analytic results are obtained by a perturbation analysis and the case of single-mode excitation is worked out in detail. Finally, in section 5, some concluding remarks are made and other possible applications such as the flow stabilization problem are mentioned.

2. Control of Duffing's Equation

Before dealing with the nonlinear beam equation (1.1), we consider the Duffing equation

$$\ddot{y} + \mu\dot{y} + \delta y + \beta y^3 = F \cos \omega t, \quad (2.1)$$

where the dot denotes the time derivative, the constants μ, δ, β and F are assumed to be positive here, and $\omega > 0$ is the forcing frequency. For small F , by perturbation analysis [7], it is known that equation (2.1) has a periodic solution of the form

$$y = A \cos(\omega t + \theta) \quad (2.2)$$

for some phase shift θ , where the amplitude A is related to the frequency ω by the response equation [8].

$$\left[(\omega^2 - \delta)A - \frac{3}{4}\beta A^3 \right]^2 + \mu\omega^2 A^2 = F^2. \quad (2.3)$$

By varying the value of F , equation (2.3) yields a family of response curves in the $|A| - \omega$ plane. Referring to Fig.1 for $F = F_0, F_1$, the solid portion of the curve corresponds to the stable regime for the periodic solution (2.2), while the dotted part of the curve (between two points of vertical tangency) renders the solution unstable. With the frequency ω fixed, point U on the F_0 -curve is unstable, but, by changing F_0 to F_1 , point U moves up to point S on the F_1 -curve becoming a stable point. Therefore, in this case by adding an in-phase force with the same frequency, an unstable periodic motion can be stabilized. On the other hand, if the forcing amplitude F is large, the system may exhibit a subharmonic response. For example, consider the case of subharmonic response with frequency $\frac{\omega}{3}$. Again, by a perturbation analysis, it is found that equation (2.1) has a subharmonic solution of the form [8]:

$$y = A \cos(\omega t + \theta_1) + B \cos\left(\frac{\omega}{3}t + \theta_2\right)$$

where A, B and θ_1, θ_2 are the corresponding amplitudes and phases, which satisfy some response equations. For the subharmonics, the equation reads

$$\omega^2 = 9\alpha + \frac{27}{4}\gamma(B^2 + 2f^2) \pm \left[\left(\frac{27\gamma B}{2} \right)^2 - \mu^2 \right]^{\frac{1}{2}} \quad (2.4)$$

with $f = 9F/8$. For $F = F_0, F_1$ with $F_1 > F_0 > 0$, the response curves associated with (2.4) are given

in Fig.2. Note that for $\omega = \omega_0$, point P on the F_0 -curve corresponds to a subharmonics with amplitude r_0 . However, this subharmonics will disappear when F changes from F_0 to F_1 , since $\omega < \omega_0$, and ω_1 is the smallest frequency for the existence of a subharmonics at $F = F_1$. This may explain qualitatively why a subharmonic vibration can be suppressed in the experimental investigation [2] by an additive periodic force, which has the effect of changing the forcing amplitude F .

In contrast with the additive vibrational control, the control can be applied parametrically. For instance we regard the Duffing equation (2.1) as an approximate equation for an inverted pendulum near the upper equilibrium position ($y = 0$), for which $\alpha = -\delta < 0$. Clearly $y = 0$ is an unstable equilibrium. If the suspension point vibrates at a high frequency $v \gg \omega$, the equation (2.1) should be replaced by [3]

$$\ddot{z} + \mu \dot{z} + [p(vt) - \delta]z + \beta z^3 = F \cos \omega t, \quad (2.5)$$

where $p(\tau) = p(\tau + 2\pi)$ is a periodic function. Without the control p , a periodic motion about $y = 0$ is obviously unstable. However, by the method of averaging [3], equation (2.5) can be closely approximated by the averaged equation (see section 4):

$$\ddot{y} + \mu \dot{y} + [\langle p^2 \rangle - \delta]y + \beta y^3 = F \cos \omega t, \quad (2.6)$$

where $\langle p \rangle = 0$ and

$$\langle p^n \rangle = \frac{1}{2\pi} \int_0^{2\pi} p^n(\tau) d\tau, \text{ for } n = 1, 2.$$

Therefore, if $\alpha = \langle p^2 \rangle - \delta > 0$, the periodic motion can now be stabilized as before.

The above examples show the possibility of stabilizing periodic motions by vibrational control. As a generalization we consider the following control problem:

$$\ddot{z} + \mu \dot{z} + \alpha z + \beta z^3 = f(\lambda, \omega t) + h(\lambda, vt, z) \quad (2.7)$$

where $f(\lambda, \tau) = f(\lambda, \tau + 2\pi)$ and $h(\lambda, \tau, z) = h(\lambda, \tau + 2\pi, z)$ are periodic functions; λ is a control parameter and h is a certain control function with $h(0, vt, y) = 0$ and ω, v are the vibration frequencies with $v \gg \omega$. The uncontrolled case corresponds to $\lambda = 0$ and $h = 0$. Of course the equations (2.1) and (2.5) are special cases of (2.7). Suppose that $z = \varphi_0(t)$ is unstable periodic solution of equation (2.7) when $\lambda = 0$ and $h = 0$. The control objective is to choose the control parameter λ and function h so that the corresponding periodic solution $z = \varphi(\lambda, t)$ with $\varphi(0, t) = \varphi_0(t)$, becomes asymptotically stable. This means analytically that the variational equation for $y = (z - \varphi)$ from (2.7) has only exponentially decaying solutions. More precisely, if

$$r(\lambda, \varphi) = \lim_{t \rightarrow \infty} \frac{1}{t} \ln |y(t)|,$$

then we must choose λ and h such that $r < 0$. Obviously, unlike the optimal control, such a control, if possible, is far from unique. The choice of λ and h , though guided by physical feasibility, is mostly up to the personal preference. In what follows, this control principle will be generalized to deal with nonlinear partial differential equations.

3. Stabilization of Nonlinear Evolutions Equations

In the theoretical discussion, it is convenient to consider the partial differential equations of interest as a nonlinear evolution equation of the form

$$\begin{cases} \frac{du}{dt} = B(u) + F(\lambda, \omega t) + H(\lambda, \nu t, u), \\ u(0) = h, \end{cases} \quad (3.1)$$

where $u(t)$ is a vector in some infinite-dimensional vector space V with initial state h . The operator B is nonlinear, $F(\lambda, \tau) = F(\lambda, \tau + 2\pi)$ and $H(\lambda, \tau, u) = H(\lambda, \tau + 2\pi, u)$ are periodic with control parameter λ , being a scalar or vector. The control function H acts parametrically with rapid oscillations so that $\nu \gg \omega$. When $\lambda = 0$, $H(0, \tau, u) = 0$ and the system (3.1) is uncontrolled. We are interested in stabilizing a periodic motion which is unstable at $\lambda = 0$. If the equilibrium solution u_0 of (3.1) at $\lambda = 0$ is also unstable, we introduce a parametric vibrational control H to stabilize it as in the case of an inverted pendulum. The effect of H can be examined by the method of averaging [3]. By a change of time from t to $\sigma = t/\varepsilon$ with $\varepsilon = 1/\nu$, the system (3.1) can be approximated by the averaged equation

$$\begin{cases} \frac{du}{dt} = B(u) + F(\lambda, \omega t) + \square(\lambda, u), \\ u(0) = h, \end{cases} \quad (3.2)$$

where

$$\square(\lambda, u) = \langle H(\lambda, \nu t, u) \rangle = \frac{1}{2\pi} \int_0^{2\pi} H(\lambda, \tau, u) d\tau. \quad (3.3)$$

The function H should be chosen so that the equilibrium solution u_1 of the averaged equation (3.2) becomes stable. Without control ($\lambda = 0$), let $u = \psi_0(t)$ be an unstable periodic solution of equation (3.1) near u_0 . In addition to the parametric control H , we have modulated the forcing function $F(\lambda, \omega t,)$ by tuning the control parameter λ so that the corresponding periodic motion satisfying the averaged equation (3.2) is asymptotically stable. To this end let us consider the variational equation of (3.2) for $v = (u - \psi)$:

$$\begin{cases} \frac{dv}{dt} = B_1(\psi, v) + H_1(\lambda, \psi, v), \\ v(0) = g, \end{cases} \quad (3.4)$$

where

$$\begin{aligned} B_1(\psi, v) &= B(v + \psi) - B(\psi), \\ H_1(\lambda, \psi, v) &= \square(\lambda, v + \psi) - \square(\lambda, \psi), \end{aligned}$$

and g is an initial vector in V . Let $\|h\|$ denote the magnitude (norm) of vector h . The control objective is then to choose function H and parameter λ in such a way that the Liapunov exponent r is negative,

$$r(\lambda, H) = \lim_{t \rightarrow \infty} \frac{1}{t} \ln \|v(t)\| < 0, \quad (3.5)$$

for all g with $\|g\| < \delta$ with some $\delta > 0$. For small δ , the variational equation (3.4) can be linearized to give

$$\begin{cases} \frac{dv}{dt} = A(\lambda, t)v, \\ v(0) = g, \end{cases} \quad (3.6)$$

where $A(\lambda, t)$ is a linear operator defined by

$$A(\lambda, t)v = \{B_u[\psi(\lambda, t)] + H_u[\lambda, \psi(\lambda, t)]\}v, \quad (3.7)$$

and $B_u(v) = \frac{\delta B(v)}{\delta v}$, $H_u(\lambda, v) = \frac{\delta H(\lambda, v)}{\delta v}$ are the linearized operator of B and H at v . Note that $A(\lambda, t)$ is periodic with the same period T as that of ψ . Now, if V is finite-dimensional and $A(\lambda, t)$ is a matrix, then, by the Floquet theory [7], the solution of equation (3.6) can be expressed as

$$v(t) = P(t)e^{tR}g, \quad (3.8)$$

where $P(t) = P(t + T)$ is a periodic matrix, and R is a constant matrix. The smallest real part of the eigenvalues of R yields the Liapunov exponent r . Of course the representation (3.4) holds for any finite-dimensional approximation of equation (3.4). Unfortunately, even the periodic function ψ is known, the analytical computation of the Liapunov exponent r through either (3.5) or (3.8) is impossible without simplifying assumptions. For example, for small amplitude vibration, the nonlinearity is weak so that the perturbation method and an eigenfunction expansion can be applied. This procedure will be illustrated in the application to the panel vibration problem.

4. Vibrational Control of Elastic Panel

By redefining the constants in the nonlinear beam equation (1.1) under a vibrational control, it yields

$$\partial_t^2 w + \mu \partial_t w - (\alpha + \beta \|\partial_x w\|^2) \partial_x w + \gamma \partial_x^4 w = p(\lambda, \omega t, x) + h(\lambda, \nu t, x, w) \quad (4.1)$$

where the initial-boundary conditions are omitted, and

$$\|\partial_x w\|^2 = \int_0^l |\partial_x w|^2 dx \quad (4.2)$$

$$p(\lambda, \omega t, x) = \Delta p(\omega t, x) + (\lambda, \omega t, x), \quad (4.3)$$

p_1 and h are the additive and parametric control forces with frequencies $\omega \gg \nu$. The physical constants μ, β, γ are positive, while α is positive or negative depending on the axial force being tensile or compressive. Without control, we assume that, at $\lambda = 0$, $p_1(0, x, \tau) = h(0, \sigma, x, w) = 0$. To be specific, we choose the parametric control to be a vibrational axial force of the form

$$h(\lambda, \nu t, x, w) = \dot{q}(\lambda, \nu t) \partial_x^2 w, \quad (4.4)$$

where

$$\dot{q} = \partial_t q(\lambda, \nu t)$$

with

$$\langle q \rangle = \langle \dot{q} \rangle = 0. \quad (4.5)$$

Now let $u_1 = w$ and u_2 defined by

$$\dot{u}_1 = u_2 + q(\lambda, \nu t) \partial_x^2 u_1 \quad (4.6)$$

Then equations (4.1) and (4.6) yield

$$\begin{aligned} \dot{u}_2 = & -\left\{ \mu(u_2 + q \partial_x^2 u_1) - (\alpha + \beta \|\partial_x u_1\|^2) \partial_x^2 u_1 + \gamma \partial_x^4 u_1 \right\} \\ & - q(\partial_x^2 u_2 - q \partial_x^4 u_1) + p(\lambda, \omega t, x). \end{aligned} \quad (4.7)$$

We set

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

and rewrite the equations (4.6) and (4.7) in the form (3.1):

$$\frac{du}{dt} = B(u) + F(\lambda, \omega t) + H(\lambda, \omega t, u), \quad (4.8)$$

where

$$B(u) = \begin{bmatrix} u_2 \\ -\left\{ \mu u_2 - (\alpha + \beta \|\partial_x u_1\|^2) \partial_x^2 u_1 + \gamma \partial_x^4 u_1 \right\} \end{bmatrix}, \quad (4.9)$$

$$\tilde{F}(\lambda, \omega t) = \begin{bmatrix} 0 \\ p(\lambda, \omega t, \bullet) \end{bmatrix}, \quad (4.10)$$

and

$$H(\lambda, \nu t, u) = \begin{bmatrix} q \partial_x^2 u_1 \\ -\mu q \partial_x^2 u_1 - q^2 \partial_x^4 u_1 - q \partial_x^2 u_2 \end{bmatrix}. \quad (4.11)$$

In view of equations (4.5) and (4.9)-(4.11), by taking the time-average of equation (4.8) in $\sigma = \nu t$ with $\tau = \omega t$ fixed, we get

$$\frac{du}{dt} = B(u) + F(\lambda, \omega t) + \square(\lambda, u), \quad (4.12)$$

where

$$\square(\lambda, u) = \begin{bmatrix} 0 \\ -\langle q^2 \rangle \partial_x^4 u_1 \end{bmatrix}. \quad (4.13)$$

We note that the average equation (4.12) yields a scalar equation for $w = u_1$ as follows,

$$\partial_t^2 w + \mu \partial_t^2 w - (\alpha + \beta \|\partial_x w\|^2) \partial_x^2 w + (\gamma + \langle q^2 \rangle) = p(\lambda, \omega t, x), \quad (4.14)$$

which shows that the high-frequency axial vibrational force $q(\lambda, \nu t)$ has the effect of increasing the bending stiffness γ by the magnitude of $\langle q^2 \rangle$. Thus it stabilizes the system statically in general. Now let

$$u_0 = \psi_0(t) = \begin{bmatrix} \psi_{01}(t) \\ \psi_{02}(t) \end{bmatrix}$$

be an unstable periodic solution of equation (4.8) when $\lambda = 0$ ($H = 0$), and let

$$u = \psi(\lambda, t) = \begin{bmatrix} \psi_1(\lambda, t) \\ \psi_2(\lambda, t) \end{bmatrix}$$

be a periodic solution of equation (4.14) with $\psi(0, t) = \psi_0(t)$. Define $v = (u - \psi)$ so that v satisfies the variational equation (3.4). Here it can be written in the form

$$\frac{dv}{dt} = A(\lambda, t)v + \beta G(\lambda, t, v), \quad (4.15)$$

where A is a periodic linear operator and G is a nonlinear mapping defined as

$$Av = \begin{bmatrix} v_2 \\ f_1 \end{bmatrix}, G = -\begin{bmatrix} 0 \\ f_2 \end{bmatrix},$$

and

$$\begin{aligned} f_1 &= -\mu v_2 + (\alpha + q^2 + \beta \|\partial_x \psi_1\|^2) \partial_x^2 v_1 + 2\beta (\partial_x \psi_1, \partial_x v_1) \partial_x^2 \psi_1 - \gamma \partial_x^4 v_1, \\ f_2 &= \|\partial_x v_1\|^2 \partial_x^2 \psi_1 + 2(\partial_x \psi_1, \partial_x v_1) \partial_x^2 \psi_1 + \|\partial_x v_1\|^2 \partial_x^2 v_1, \end{aligned}$$

with the inner product notation

$$(g, h) = \int_0^l g(x)h(x)dx.$$

When the nonlinear term G is dropped, equation (4.15) yields a generalized Hill's equation, a linear partial differential equation with periodic coefficient:

$$\frac{dv}{dt} = A(\lambda, t)v. \quad (4.16)$$

For computational purposes, introduce a complete set of orthonormal functions $\{e_n\}$, which may be the eigenfunctions associated with the linearized problem, or $e_n(x) = \sqrt{\frac{2}{l}} \sin \frac{n\pi}{l}x$, $n = 1, 2, \dots$. By the expansion of the solutions of (4.12) and (4.15) into terms of e_n 's as follows,

$$u = \sum_{n=1}^{\infty} u_n(t)e_n; \quad v = \sum_{n=1}^{\infty} v_n(t)e_n,$$

their coefficients satisfy the infinite systems of coupled ordinary differential equations of the form:

$$\begin{aligned} \frac{du_i}{dt} &= B_i(u_1, \dots, u_n, \dots) + F_i(\lambda, t) + \square_i(\lambda, u_1, \dots, u_n, \dots), \\ \frac{dv_i}{dt} &= \sum_{n=1}^{\infty} a_{ij}(\lambda, t)v_j, \quad i = 1, 2, \dots, n, \dots, \end{aligned}$$

where $a_{ij} = (Ae_i, e_j)$. The above systems can only be solved numerically for truncated systems of

low dimensions. As mentioned before, if the nonlinear effect is weak, we can apply the perturbation analysis to approximate solutions analytically. To this end let us assume that the damping coefficient and the forcing amplitude are small. By proper scaling with respect to a small parameter $\varepsilon > 0$, the equation (4.14) is rewritten as

$$\partial_t^2 w + \varepsilon \mu \partial_t w - (\alpha + \varepsilon \beta \|\partial_x w\|^2) \partial_x^2 w + \gamma \partial_x^4 w = \varepsilon p(\lambda, \omega t, x), \quad (4.17)$$

for which we assume $\alpha > 0$ and set $\gamma = 0$ for simplicity. It remains to study the problem of additive control. To illustrate the perturbation procedure, we will analyze the case of single-mode excitation in some detail.

Let us consider the case of n th mode harmonic excitation in (4.17):

$$p(\lambda, \omega t, x) = F(\lambda) \sin \frac{n\pi}{l} x \cos \omega t, n = 1, 2, \dots \quad (4.18)$$

where the control parameter λ modulates the forcing amplitude F and $F_0 = F(0)$ is the uncontrolled amplitude. Then the equation (4.17) admits a single-mode solution

$$w = z_n(t) \sin \frac{n\pi}{l} x \quad (4.19)$$

and z_n satisfies the Duffing equation:

$$\ddot{z}_n + \varepsilon \mu \dot{z}_n + \alpha_n z_n + \varepsilon \beta_n z_n^3 = \varepsilon F(\lambda) \cos \omega t, \quad (4.20)$$

where

$$\alpha_n = \left[\alpha + \gamma \left(\frac{n\pi}{l} \right)^2 \right] \left(\frac{n\pi}{l} \right)^2, \\ \beta_n = \frac{\beta l}{2} \left(\frac{n\pi}{l} \right)^4.$$

The perturbation analysis of Duffing's equation has been discussed by many authors (see e.g. [3], [7]). Here we adopt the method of averaging by letting

$$z_n = y_1(t) \sin \omega t + y_2(t) \cos \omega t, \quad (4.21)$$

with

$$\dot{y}_1 \sin \omega t + \dot{y}_2 \cos \omega t = 0,$$

which are substituted into (4.20) to give

$$\begin{cases} \dot{y}_1 = \frac{1}{2\omega} \left[\delta_n y_2 - \frac{3}{4} \beta_n |y|^2 y_2 + \mu \omega y_1 + F(\lambda) \right], \\ \dot{y}_2 = \frac{-1}{2\omega} \left[\delta_n y_1 - \frac{3}{4} \beta_n |y|^2 y_1 + \mu \omega y_2 \right], \end{cases} \quad (4.22)$$

with $\delta_n = (\omega^2 - \alpha_n)$ and $|y|^2 = y_1^2 + y_2^2$. In the polar form, $y_1 = r \sin \varphi, y_2 = r \cos \varphi$, this equation becomes

$$\begin{cases} \dot{r} = \frac{1}{2\omega} R(r, \varphi, \omega), \\ \dot{\varphi} = \frac{1}{2\omega} \Phi(r, \varphi, \omega), \end{cases} \quad (4.23)$$

where

$$\begin{cases} R = \mu\omega r - F \sin \varphi \\ \Phi = \delta_n - \frac{3}{4}\beta_n r^2 + \frac{F}{r} \cos \varphi. \end{cases} \quad (4.24)$$

The solution (4.21) can be written as

$$z_n = r(t) \cos[\omega t + \varphi(t)]. \quad (4.25)$$

Therefore, for z_n being periodic with frequency ω , r and φ must be constants, which correspond to the equilibrium point $(\tilde{r}, \tilde{\varphi})$ of equation (4.23) satisfying

$$\begin{cases} R(\tilde{r}, \tilde{\varphi}, \omega) = 0, \\ \Phi(\tilde{r}, \tilde{\varphi}, \omega) = 0. \end{cases} \quad (4.26)$$

By taking equation (4.24) into account, the above equation can be solved approximately to give

$$\left(\delta_n \tilde{r} - \frac{3}{4}\beta_n \tilde{r}^3\right)^2 + \mu \tilde{r}^2 \omega^2 = F^2(\lambda), \quad (4.27)$$

which, by a change of notation, agrees with the response relation (2.3). Therefore for each n , the response curves are shown in Fig.1. Schematically, for $n = 1, 2, \dots$, the response curves are plotted in Fig.3. Geometrically the control strategy is to steer an unstable point U_n on the F_0 -curve to a stable point S_n on the F_1 -curve. Analytically the stability of a periodic solution is now reduced to that of an equilibrium point, which can be checked more easily. To do so we form the first variational equation of (4.23) about $(\tilde{r}, \tilde{\varphi}) =$

$$\begin{cases} \dot{\rho} = \frac{1}{2\omega} (\tilde{R}_r \rho + \tilde{R}_\varphi \tilde{\Phi}_\varphi), \\ \dot{\vartheta} = \frac{1}{2\omega} (\tilde{\Phi}_r \rho + \tilde{\Phi}_\varphi \tilde{\Phi}_r), \end{cases} \quad (4.28)$$

where $\tilde{R}_r = \partial_r R(\tilde{r}, \tilde{\varphi}, \omega)$, $\tilde{R}_\varphi = \partial_\varphi R(\tilde{r}, \tilde{\varphi}, \omega)$ and so on. Let $\eta(\lambda)$ denote an eigenvalue of the coefficient matrix of (4.28). It can be readily verified that, by making use of (4.24) and (4.27), if

$$D(\lambda) = (\tilde{R}_r \tilde{\Phi}_\varphi - \tilde{R}_\varphi \tilde{\Phi}_r) > 0, \quad (4.29)$$

then $\text{Re } \eta(\lambda) < 0$ so that the steady state $(\tilde{r}, \tilde{\varphi})$ is stable. This is of course the stability condition for the associated periodic solution. After computing the partial derivatives in (4.29), it yields

$$D(\lambda) = \left(\frac{3}{4}\beta_n \tilde{r}^2 - \delta_n\right) \left(\frac{9}{4}\beta_n \tilde{r}^2 - \delta_n\right) + \mu^2 \omega^2 > 0, \quad (4.30)$$

with $\delta_n = (\omega^2 - \alpha_n)$. The above inequality determines the stability regime S in the $\tilde{r} - \omega$ plane. In view of (4.27), $\tilde{r}(\lambda)$ depends on the control parameter λ , which will take an unstable point into the stable regime S . Note that from (4.30), we can get a simple sufficient stability condition:

$$\omega^2 < \alpha_n + \frac{3}{4}\beta_n \tilde{r}^2 \lambda, \quad (4.31)$$

or

$$\omega^2 > \alpha_n + \frac{9}{4}\beta_n \tilde{r}^2 \lambda.$$

The above inequalities give rise to stable (shaded) sub-regions as shown in Fig.4.

In general all modes are excited by periodic pressure fluctuations. For instance, consider the harmonic forcing (4.18) with a general spatially dependent amplitude

$$p(\lambda, \omega t, x) = F(\lambda, x) \cos \omega t. \quad (4.32)$$

If the axial load is compressive ($\alpha < 0$) and slightly exceeds the lowest buckling load $\gamma \left(\frac{\pi}{l} \right)^4$, the parametric control $q(vt)$ can still be used to stabilize the system statically by choosing $\langle q^2 \rangle \gg |\alpha|$. So we remain to consider equation (4.17) by assuming $\alpha > 0$ there. To apply the above perturbation procedure, we need to expand w in (4.17) and F in (4.32) into infinite series with respect to the modal function $e_n(x) = \sqrt{\frac{2}{l}} \sin \frac{n\pi}{l} x$, for $n = 1, 2, \dots$. The resulting infinite system of coupled nonlinear differential equations for the coefficient functions can then be treated by a perturbation analysis. Such a procedure developed previously for nonlinear wave equations by one of us (Chow [9]) can be applied here. However unlike the single mode situation, simple stability conditions such as (4.30) or (4.31) are no longer attainable. Though it is possible to study the stability regime numerically after a finite-mode approximation, this has not yet been done.

5. Concluding Remarks.

In the paper we present a general method of vibrational control for a certain class of nonlinear evolution equations with a particular reference to the nonlinear beam equation arising from the panel structure dynamics. The control consists of a high frequency parametric vibration and the forcing amplitude modulation. The high-frequency control is to affect a change in system parameter for static stability, while the additive control of the excitation force, if needed, is to stabilize an unstable periodic motion. In application to the panel structure, we show that, for a periodically excited panel near a buckled state, a high frequency oscillatory axial force can keep the system in the state of periodic motion, which can then be stabilized by an additive force modulation. The reason that we only control the force amplitude, instead of both the amplitude and phase is that the additive control is the most effective when it is in phase or out of phase with the excitation force. For a small forcing amplitude, a perturbation technique can be used to reduce the stabilization of a periodic motion to that of an equilibrium point, the latter of which is much simpler to analyze. In the case of a single-modal excitation, an explicit stability condition is obtained. By a finite-modal approximation, the stabilization problem can be studied numerically but has not yet been treated. The vibrational control principle described in this paper can also applied to other problems such as the flow stability control. Here the nonlinear evolution equation is given by the Navier-Stokes equation. For a slightly unstable flow, the perturbation analysis by Keller and Kogelman [10] can be employed to deal with the flow stabilization by vibrational control.

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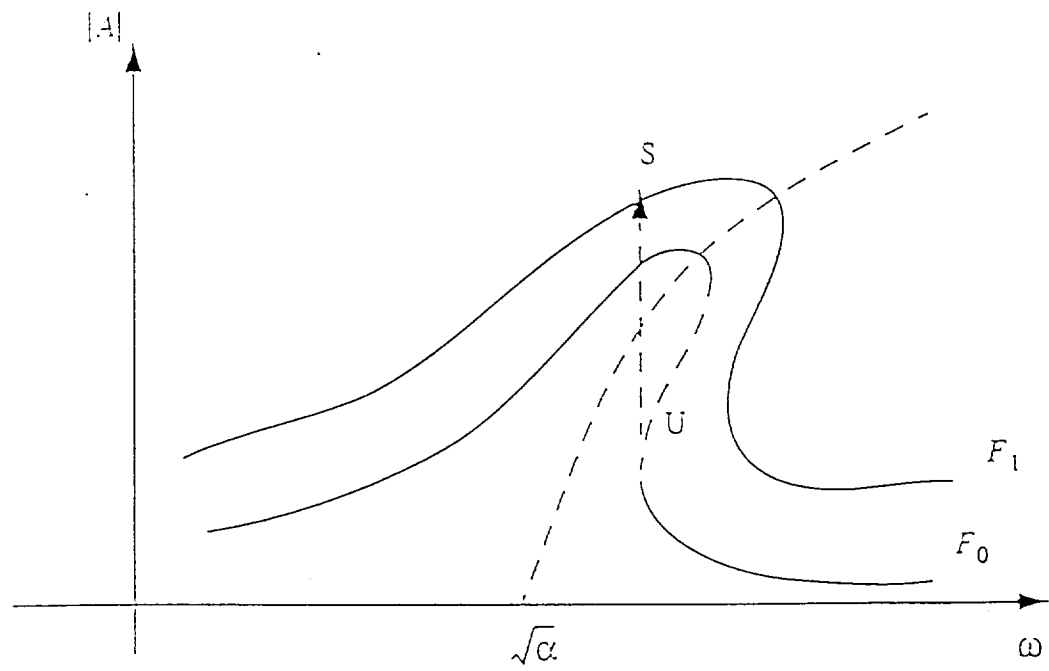


Fig. 1: Response Curves for Harmonic Oscillation

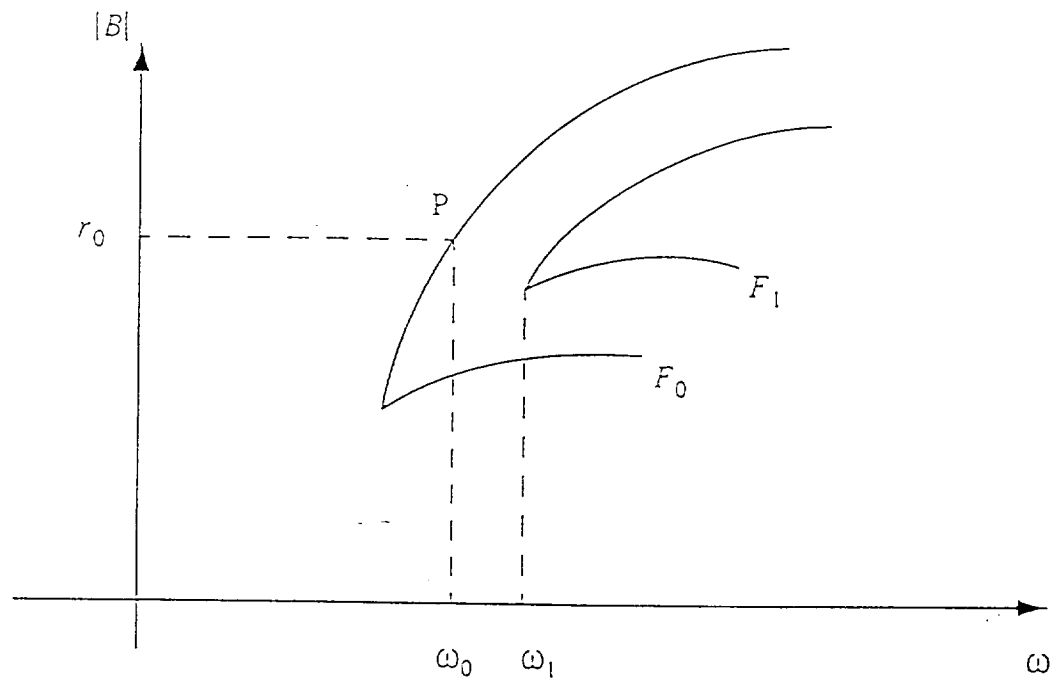


Fig. 2: Response Curves for Subharmonic Oscillation

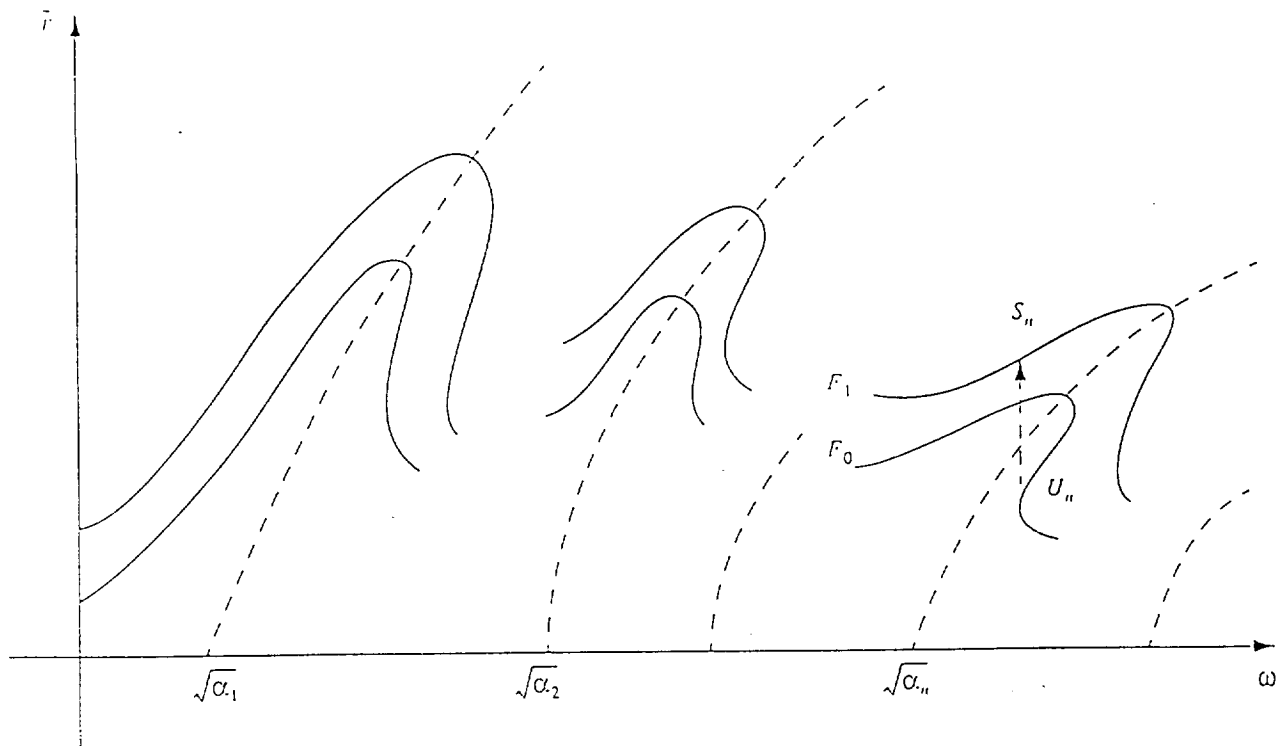


Fig. 3: Response Curves for Different Modes of Excitations

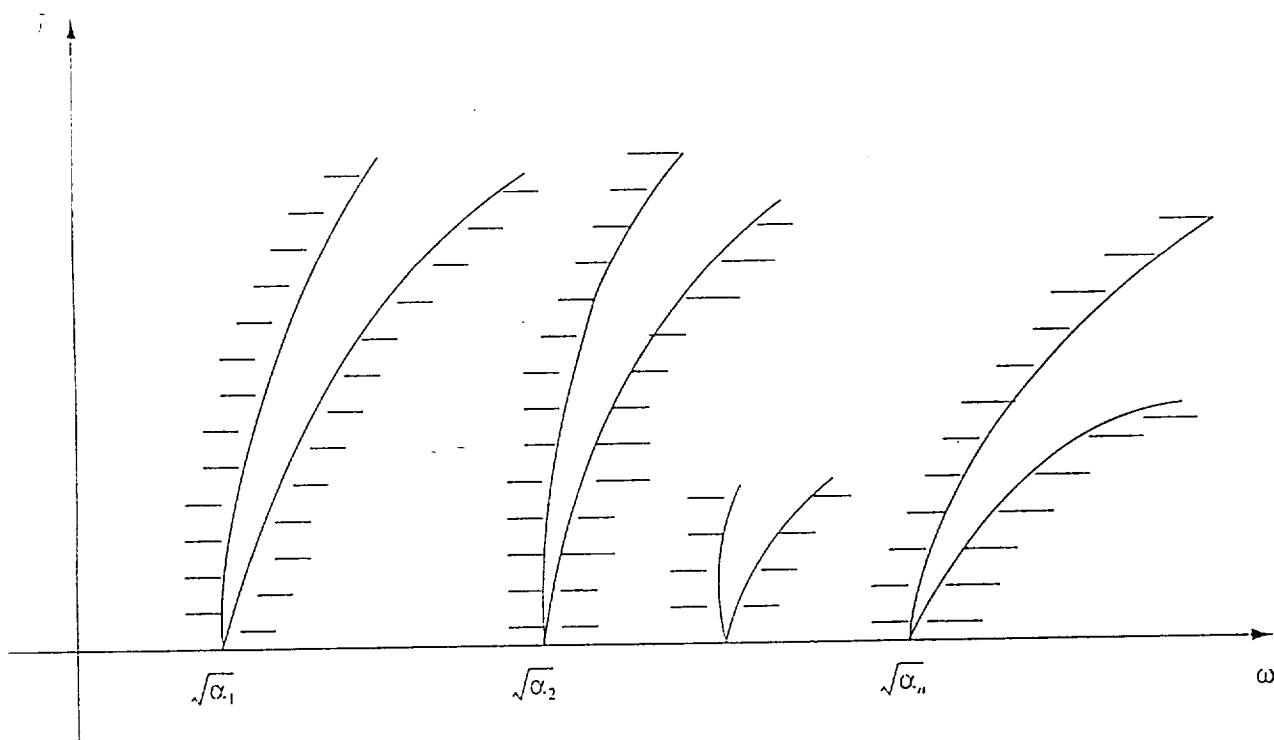


Fig. 4: Stability Regions (Shaded) for Different Modes of Excitations